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Vector Algebra→RACE-1

Q.1- Classify the following measures as scalar and vector quantities :
(i) Mass (ii) Work (iii) Force (iv) Current

Q.2- Represent graphically a displacement of 30 km, 30° west of north after representation of vectors.

Q.3- If A is the point (1, 2) in x–y plane and the vector $\overline{AB}$ has components 2 and 6, find the point B.

Q.4- Find the direction cosines of the ray from P to Q where P is point (1, –2, 2) and Q is the point (3, –5, –4).

Q.5- Write the direction ratios of the vector $\vec{a} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and hence calculate its direction cosines.

Q.6- Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with y and z-axes, respectively.

Q.7- Find a unit vector in the direction of $\vec{a} = 12\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$.

Q.8- If $\overrightarrow{V_1} = 2\hat{\imath} - 3\hat{\jmath}$, $\overrightarrow{V_2} = \hat{\jmath}$ and $\overrightarrow{V_3} = \hat{\imath} - 6\hat{\jmath}$ find a unit vector. In the direction of
(i) $\overrightarrow{V_1} + 2\overrightarrow{V_2} - \overrightarrow{V_3}$ (ii) parallel to $\overrightarrow{V_1} + 2\overrightarrow{V_2} - \overrightarrow{V_3}$

Q.9- Show that the points A(3, – 4, 5), B(2, 1, 7), C(6, –19, –1) are collinear.

Q.10- Prove that the points with position vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ & $\vec{c} = -7\hat{\jmath} + 10\hat{k}$ are collinear.

Q.11- Find the values of x and y if (x, –1, 3), (3, y, 1) and (–1, 11, 9) are collinear.

Q.12- Find the sum of the vectors $\vec{a} = 4\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$, $\vec{b} = -3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{c} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$.

Q.13- Find the vector joining the points A(4, 5, 1) and B(–3, –4, –6) directed from A to B.

Q.14- Find the position vector of the points which divide internally and externally in the ratio. 2 : 3, the segment joining the points with P.V. $2\vec{a} - 3\vec{b}$ and $3\vec{a} - 2\vec{b}$.

Q.15- If $\vec{a}$ and $\vec{b}$ are the position vectors of point A and B respectively, find the position vector of a point C in BA produce such that BC = 1.5 BA.

Q.16- Find the angle between the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $3\hat{\imath} + 4\hat{\jmath} - \hat{k}$.

Q.17- Show that the vectors $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$, $\vec{c} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$ form a right-angled triangle.

Q.18- If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$, find λ such that $\vec{a}$ is perpendicular to $\vec{\lambda}\vec{b} + \vec{c}$.

Q.19- If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$, show that the angle between $\vec{a}$ and $\vec{b}$ is 60°.

Q.20- The projection of $\overrightarrow{PQ}$ on $\overrightarrow{AB}$ where $\overrightarrow{PQ} = 5\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\overrightarrow{AB} = 3\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ will be.

Q.21- The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

Q.22- Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $-\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$.

Q.23- If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

Vector Algebra→RACE-2

Q.1- Show that the points A, B, C with position vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

Q.2- Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

Q.3- If θ is the angle between two vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ find sinθ.

Q.4- Let $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = \widehat{3\imath} + \hat{\jmath} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}.\vec{a} = 21$.

Q.5- (i) Show that the points $A(2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$, $B(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$and $C(7\hat{\imath} - \hat{k})$are collinear.

(ii) Find $|\vec{a} \times \vec{b}|$if $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$, and $\vec{b} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$.

Q.6- The scalar product of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to 1 find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

Q.7- If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then $|\lambda\vec{a}|$ lies in.

Q.8- The area of a triangle formed by vertices O, A and B, where $\overrightarrow{OA} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ is

(A) $3\sqrt{5}$sq. units (B) $5\sqrt{5}$sq. units (C) $6\sqrt{5}$sq. units (D) 4 sq. units

Q.9- (i) Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

(ii) Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\vec{a} - \vec{b}$ is also a unit vector.

Q.10- Find the angle between the vector $(\hat{\imath} - \hat{\jmath})$ and $(\hat{\jmath} - \hat{k})$.

Q.11- If $\vec{a} = \alpha\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$and $\vec{b} = 2\hat{\imath} - \hat{\jmath} - \beta\hat{k}$. Find the value of α and β. So that $\vec{a}$ and $\vec{b}$ may be collinear.

Q.12- Find the magnitude of vector $\vec{a}$ given by $\vec{a} = (\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) \times (-\hat{\imath} + 3\hat{k})$.

Q.13- If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors, find the value of $|\vec{a} + 2\vec{b} + 3\vec{c}|$.

Q.14- If the side AB and BC of a parallelogram ABCD are represented as vectors $\overrightarrow{AB} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\overrightarrow{BC} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, then find the unit vector along diagonal AC.

Q.15- Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector $\vec{a}$ where $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$.

Q.16- The two adjacent sides of a parallelogram are represented by vectors $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram.

Q.17- (i) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \ne 0$ then show that $\vec{b} = \vec{c}$.

(ii) If $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then find the value of $(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$.

Vector Algebra→RACE-3

Q.1- The unit vector in the direction of vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ is:

(A) $\frac{1}{\sqrt{3}}\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)$ (B) $\sqrt{3}\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)$ (C) $\frac{1}{\sqrt{2}}\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)$ (B) $\sqrt{2}\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)$

Q.2- Projection of vector $\vec{a}$ on $\vec{b}$ is:

(A) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ (B) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (C) $\frac{\vec{a}}{|\vec{a}|}$ (D) $\frac{\vec{a}\times\vec{b}}{|\vec{b}|}$

Q.3- L and M are two points with position vector $2\vec{a}-\vec{b}$ and $\vec{a}+2\vec{b}$ respectively. The position vector of a point N which divides the line segment LM in the ratio 2 : 1 externally is.

Q.4- If $\vec{a}=3\hat{\imath}-\hat{\jmath}-4\hat{k}\ \vec{b}=-2\hat{\imath}+4\hat{\jmath}-3\hat{k}$; then the magnitude of vector $3\vec{a}-2\vec{b}$ is:

(A) 18 (B) 17 (C) $\sqrt{326}$ (D) 19

Q.5- The angle between the two vectors $\vec{a}=2\hat{\imath}-\hat{\jmath}-3\hat{k}$ and $\vec{b}=3\hat{\imath}-2\hat{\jmath}-2\hat{k}$is:

(A) 30° (B) 60° (C) $cos^{-1}\left(\frac{10}{\sqrt{238}}\right)$ (D) 90°

Q.6- The sum of three vectors represented by the consecutive sides of a triangle is:

(A) $0\hat{\imath}+0\hat{\jmath}+0\hat{k}$ (B) $1\hat{\imath}+\hat{\jmath}+\hat{k}$ (C) $2\hat{\imath}+\hat{\jmath}-\hat{k}$ (D) $4\hat{\imath}-\hat{\jmath}+\hat{k}$

Q.7- If $\overrightarrow{OP}=\hat{\imath}+4\hat{\jmath}-3\hat{k}$ and $\overrightarrow{OQ}=2\hat{\imath}-2\hat{\jmath}-\hat{k}$ then $|\overline{PQ}|$ is:

(A) $\sqrt{40}$ (B) $\sqrt{41}$ (C) $\sqrt{42}$ (D) $\sqrt{43}$

Q.8- The vector $cos\ \alpha\ cos\beta\hat{\imath}+cos\alpha\ sin\beta\hat{\jmath}+sin\alpha\hat{k}$ is a:

(A) null vector (B) unit vector (C) constant vector (D) none of these

Q.9- If OACB is a parallelogram with $\overrightarrow{OC}=\vec{a}$ and $\overrightarrow{AB}=\vec{b}$ then $\overrightarrow{OA}$ is:

(A) $\vec{a}+\vec{b}$ (B) $\vec{a}-\vec{b}$ (C) $\frac{1}{2}(\vec{b}-\vec{a})$ (D) $\frac{1}{2}(\vec{a}-\vec{b})$

Q.10- If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ, then the value of $\left|\vec{a}-\vec{b}\right|$is:

(A) $2sin\frac{\theta}{2}$ (B) $2sin\theta$ (C) $2cos\frac{\theta}{2}$ (D) $2cos\theta$

Case Study

Q.11- A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors.

Based on the given information, answer the following questions.

(i) If $\vec{a}=\hat{\imath}-2\hat{\jmath}$, $\vec{b}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$; then evaluate $\left(2\vec{a}+\vec{b}\right).\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$

(ii) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}.\vec{b}=\vec{a}$, $\vec{c}=0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$; then find $\vec{a}$?

Vector Algebra→RACE-4

Q.1- Find the vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

Q.2- Let the vector $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

Q.3- Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. If $(\vec{a} + \vec{b})$is a unit vector, then find θ.

Q.4- If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$ and if $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then θ is:

Q.5- Find $|\vec{a}|$ and $|\vec{b}|$, If $(\vec{a} + \vec{b})(\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.

Q.6- Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c} = 18$.

Q.7- If a unit vector $\vec{a}$ makes angle $\frac{\pi}{3}$ with $\hat{\imath}$, $\frac{\pi}{4}$ with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$.

Q.8- If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$?

Q.9- If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$, and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$: find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

Q.10- Show that the points A, B, C with position vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ respectively are the vertices of a right angled trianlgle. Hence find the area of the triangle.

Q.11- Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels. A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture , suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters $P_1(6, 8, 4)$, $P_2(21, 8, 4)$, $P_3(21, 16, 10)$ and P4 (6,16,10); where $\vec{A}$ =P.V. of P_2 – P.V. of P_1 and $\vec{B}$ =P.V. by P_4 –P.V. of P_1 (where P.V. stands for position vector).

Based on the given information, answer the following questions.

(i) What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?

(ii) What is the magnitude of $\vec{N}$ and its units? The sun is located along the unit vector $\vec{S} = \frac{1}{2}\hat{\imath} - \frac{6}{7}\hat{\jmath} + \frac{1}{7}\hat{k}$. If the flow of solar energy is given by the vector $\vec{F} = 910\vec{S} = 910\, S$ in units of watts / meter^2 , what is the dot product of vectors $\vec{F}$with $\vec{N}$ and the units for this quantity ?

Vector Algebra→RACE-5

Q.1- Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} + 3\hat{k}$.

Q.2- Find a vector of magnitude 11 in the direction opposite to that of $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 2) and (–1, 0, 8), respectively.

Q.3- Find the position vector of a point R which divides the line joining the two points P and Q with position vectors $\overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\overrightarrow{OQ} = \vec{a} - 2\vec{b}$, respectively, in the ratio 1:2, (i) internally and (ii) externally.

Q.4- If the points (–1, –1, 2), (2, m, 5) and (3, 11, 6) are collinear, find the value of m.

Q.5- Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with y and z –axes, respectively.

Q.6- If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$, find λ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$.

Q.7- Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $-\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$.

Q.8- Using vectors, prove that cos (A–B) = cosA cosB + sinA sinB.

Q.9- Prove that in a ΔABC, $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$, where a, b, c represent the magnitude of the sides opposite to vertices A,B,C, respectively.

Q.10- The magnitude of the vector $6\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is

(A) 5 (B) 7 (C) 12 (D) 1

Q.11- The position vector of the point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ in the ratio 1 : 2 is

(A) $\frac{3\vec{a}+2\vec{b}}{3}$ (B) $\vec{a}$ (C) $\frac{5\vec{a}-\vec{b}}{3}$ (D) $\frac{4\vec{a}+\vec{b}}{3}$

Q.12- The vector with initial point P (2, –3, 5) and terminal point Q(3, –4, 7) is

(A) $\hat{\imath} - \hat{\jmath} + 2\hat{k}$ (B) $5\hat{\imath} - 7\hat{\jmath} + 12\hat{k}$ (C) $-\hat{\imath} + \hat{\jmath} - 2\hat{k}$ (D) None of these

Q.13- The angle between the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$ is

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{-\pi}{3}$ (D) $\frac{5\pi}{6}$

Q.14- The value of λ for which the two vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ are perpendicular is

(A) 2 (B) 4 (C) 6 (D) 8

Q.15- The area of the parallelogram whose adjacent sides are $\hat{\imath} + \hat{k}$ and $2\hat{\imath} + \hat{\jmath} + \hat{k}$ is

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 3 (D) 4

Q.16- If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then value of $\vec{a}.\vec{b}$ is

(A) $6\sqrt{3}$ (B) $8\sqrt{3}$ (C) $12\sqrt{3}$ (D) None of these

Q.17- The 2 vectors $\hat{\jmath} + \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC. The length of the median through A is

(A) $\frac{\sqrt{34}}{2}$ (B) $\frac{\sqrt{48}}{2}$ (C) $\sqrt{18}$ (D) None of these

Q.18- The projection of vector $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ along $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ is

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) 2 (D) $\sqrt{6}$

Q.19- If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\vec{a} - \vec{b}$ to be a unit vector?

(A) 30° (B) 45° (C) 60° (D) 90°

Q.20- The unit vector perpendicular to the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\imath} + \hat{\jmath}$ forming a right handed system is

(A) $\hat{k}$ (B) $-\hat{k}$ (C) $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}}$ (D) $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

Q.21- If $|\vec{a}| = 3$ and $-1 \le k \le 2$, then $|k\vec{a}|$ lies in the interval

(A) [0, 6] (B) [–3, 6] (C) [3, 6] (D) [1, 2]

Vector Algebra→RACE-6

Short Answer (S.A.)

Q.1- Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\jmath} + \hat{k}$.

Q.2- If $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$, find the unit vector in the direction of

(i) $6\vec{b}$ (ii) $2\vec{a} - \vec{b}$

Q.3- Find a unit vector in the direction of $\overrightarrow{PQ}$, where P and Q have co-ordinates (5, 0, 8) and (3, 3, 2), respectively.

Q.4- If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B, respectively, find the position vector of a point C in BA produced such that BC = 1.5 BA.

Q.5- Using vectors, find the value of k such that the points (k, –10, 3), (1, –1, 3) and (3, 5, 3) are collinear.

Q.6- A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, find $\vec{r}$.

Q.7- A vector $\vec{r}$ has magnitude 14 and direction ratios 2, 3, –6. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with x–axis.

Q.8- Find a vector of magnitude 6, which is perpendicular to both the vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $4\hat{\imath} - \hat{\jmath} + 3\hat{k}$.

Q.9- Find the angle between the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $3\hat{\imath} + 4\hat{\jmath} - \hat{k}$.

Q.10- If $\vec{a} + \vec{b} + \vec{c} = 0$, show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. Interpret the result geometrically?

Q.11- Find the sine of the angle between the vectors $\vec{a} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$.

Q.12- If A, B, C, D are the points with position vectors $\hat{\imath} + \hat{\jmath} - \hat{k}$, $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$, $2\hat{\imath} - 3\hat{k}$, $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, respectively, find the projection of $\overrightarrow{AB}$ along $\overrightarrow{CD}$.

Q.13- Using vectors, find the area of the triangle ABC with vertices A (1, 2, 3), B(2, –1, 4) and C(4, 5, –1).

Q.14- Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.

Long Answer (L.A.)

Q.15- Prove that in any triangle ABC, $cosA = \frac{b^2+c^2-a^2}{2bc}$, where a, b, c are the magnitudes of the sides opposite to the vertices A, B, C, respectively.

Q.16- If $\vec{a}, \vec{b}, \vec{c}$ determine the vertices of a triangle, show that $\frac{1}{2}\left[\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}\right]$ gives the vector area of the triangle. Hence deduce the condition that the three points $\vec{a}, \vec{b}, \vec{c}$ are collinear. Also find the unit vector normal to the plane of the triangle.

Q.17- Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{a} \times \vec{b}|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

Q.18- If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

Objective Type Questions

Choose the correct answer from the given four options in each of the Exercises from 19 to 33 (M.C.Q.)

Q.19- The vector in the direction of the vector $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has magnitude 9 is

(A) $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ (B) $\frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3}$

(C) $3(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ (D) $9(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

Q.20- The position vector of the point which divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3 : 1 is

(A) $\frac{3\vec{a}-2\vec{b}}{2}$ (B) $\frac{7\vec{a}-8\vec{b}}{4}$ (C) $\frac{3\vec{a}}{4}$ (D) $\frac{5\vec{a}}{4}$

Q.21- The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is

(A) $-\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$ (B) $5\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ (C) $-5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ (D) $\hat{\imath} + \hat{\jmath} + \hat{k}$

Q.22- The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a}.\vec{b} = 2\sqrt{3}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{5\pi}{2}$

Q.23- Find the value of λ such that the vectors $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ are orthogonal.

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) $-\frac{5}{2}$

Q.24- The value of λ for which the vectors $3\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $2\hat{\imath} - 4\hat{\jmath} + \lambda\hat{k}$ are parallel is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{2}{5}$

Q.25- The vectors from origin to the points A and B are $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$, respectively, then the area of triangle OAB is

(A) 340 (B) $\sqrt{25}$ (C) $\sqrt{229}$ (D) $\frac{1}{2}\sqrt{229}$

Q.26- For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{\imath})^2 \times (\vec{a} \times \hat{\jmath})^2 + (\vec{a} \times \hat{k})^2$ is equal to

(A) $\vec{a}^2$ (B) $3\vec{a}^2$ (C) $4\vec{a}^2$ (D) $2\vec{a}^2$

Q.27- If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = 12$, then value of $|\vec{a} \times \vec{b}|$ is

(A) 5 (B) 10 (C) 14 (D) 16

Q.28- The vectors $\lambda\hat{\imath} + \hat{\jmath} + 2\hat{k}, \hat{\imath} + \lambda\hat{\jmath} - \hat{k}$ and $2\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$ are coplanar if

(A) $\lambda = -2$ (B) $\lambda = 0$ (C) $\lambda = 1$ (D) $\lambda = -1$

Q.29- If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ is

(A) 1 (B) 3 (C) $-\frac{3}{2}$ (D) None of these

Q.30- Projection vector of $\vec{a}$ on $\vec{b}$ is

(A) $\left[\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right]\vec{b}$ (B) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ (C) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (D) $\left[\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right]\hat{b}$

Q.31- If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 5$, then value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}\,.\vec{a}$ is

(A) 0 (B) 1 (C) –19 (D) 38

Q.32- If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then the range of $|\lambda\vec{a}|$is

(A) [0, 8] (B) [–12, 8] (C) [0, 12] (D) [8, 12]

Q.33- The number of vectors of unit length perpendicular of the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$ is

(A) one (B) two (C) three (D) infinite

Fill in the blanks in each of the Exercises from 34 to 40.

Q.34- The vector $\vec{a} + \vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$ if ________.

Q.35- If $\vec{r}.\vec{a} = 0, \vec{r}.\vec{b} = 0$, and $\vec{r}.\vec{c} = 0$ for some non-zero vector $\vec{r}$, then the value of $\vec{a}.(\vec{b} \times \vec{c})$ is _____________

Q.36- The vectors $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + 2\hat{k}$ are the adjacent of a parallelogram. The acute angle between its diagonal is __________________.

Q.37- The values of k for which $|k\vec{a}| < |\vec{a}|$ and $k\vec{a} + \frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true are ________.

Q.38- The value of the expression $|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2$ is ____________.

Q.39- If $|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to ____________.

Q.40- If $\vec{a}$ is any non-zero vector, then $(\vec{a}.\hat{\imath})\hat{\imath} + (\vec{a}.\hat{\jmath})\hat{\jmath} + (\vec{a}.\hat{k})\hat{k}$ equals __________.

State True or False in each of the following Exercises.

Q.41- If $|\vec{a}| = |\vec{b}|$, then necessarily it implies $\vec{a} = \pm\vec{b}$.

Q.42- Position vector of a point P is a vector whose initial point is origin.

Q.43- If $|\vec{a} + \vec{b}| = |\vec{a}| - \vec{b}$, then the vectors $\vec{a}$ and $\vec{b}$ are orthogonal.

Q.44- The formula $(\vec{a} + \vec{b})^2 = \vec{a}^2 + \vec{b}^2 + 2\vec{a} \times \vec{b}$ is valid for non-zero vectors $\vec{a}$ and $\vec{b}$.

Q.45- If $\vec{a}$ and $\vec{b}$ are adjacent sides of a rhombus, then $\vec{a}.\vec{b}$=0.

3D→RACE-1

Q.1- If a line makes angles 120°, 60° and 45° with the x, y and z axis respectively, find its direction cosines.

Q.2- Find the direction cosines of the ray from P to Q where P is point (1,–2,2) and Q is the point (3,–5,–4).

Q.3- Find the vector equation for the line passing through the points (6, 4, –2) and (2, 3, 5).

Q.4- The cartesian equation of a line is $\frac{x-5}{2}=\frac{y+3}{4}=\frac{z+6}{2}$. Find the vector equation for the line.

Q.5- Find the angle between the lines
$\vec{r}=2\hat{\imath}+3\hat{\jmath}-4\hat{k}+\lambda(2\hat{\imath}+1\hat{\jmath}+2\hat{k})$ and $\vec{r}=2\hat{\imath}-5\hat{k}+\mu(6\hat{\imath}+3\hat{\jmath}+2\hat{k})$

Q.6- Find the angle between the lines whose direction cosines are given by the equations $3\iota+m+5n=0$, $6mn-2n\iota+5\iota m=0$.

Q.7- Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angles of $\frac{\pi}{3}$.

Q.8- Find the shortest distance between the lines $\vec{r}=(4\hat{\imath}-\hat{\jmath})+\lambda(\hat{\imath}+2\hat{\jmath}-3\hat{k})$ and $\vec{r}=(\hat{\imath}-\hat{\jmath}+2\hat{k})+\mu(2\hat{\imath}+4\hat{\jmath}-5\hat{k})$.

Q.9- Find the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. Also find the equations of the shortest distance.

Q.10- Find the distance of the point (2, 4, –1) from the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.

3D→RACE-2

Q.1- The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, –2) is 4. Find its z-coordinate.

Q.2- Find the shortest distance between the lines $\vec{r}=(4\hat{\imath}-\hat{\jmath})+\lambda(\hat{\imath}+2\hat{\jmath}-3\hat{k})$ and $\vec{r}=(\hat{\imath}-\hat{\jmath}+2\hat{k})+\mu(2\hat{\imath}+4\hat{\jmath}-5\hat{k})$

Q.3- Find the direction cosines of a line which makes equal angles with the coordinate axes.

Q.4- A line passes through the point with position vector $2\hat{\imath}-\hat{\jmath}+4\hat{k}$ and is in the direction of the vector $\hat{\imath}+\hat{\jmath}-2\hat{k}$. Find the equation of the line in Cartesian form.

Q.5- The coordinates of the foot of the perpendicular drawn from the point (2, –3, 4) on the y-axis is

(A) (2, 3, 4) (B) (–2, –3, –4) (C) (0, –3, 0) (D) (2, 0, 4)

Q.6- Find the shortest distance between the lines
$\vec{r}=2\hat{\imath}-\hat{\jmath}+\hat{k}+\lambda(3\hat{\imath}-2\hat{\jmath}+5\hat{k})$
$\vec{r}=3\hat{\imath}+2\hat{\jmath}-4\hat{k}+\mu(4\hat{\imath}-\hat{\jmath}+3\hat{k})$

Q.7- Find the shortest distance between the following lines :
$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

Q.8- Show that the lines :
$\frac{1-x}{2}=\frac{y-3}{4}=\frac{z}{-1}$ and $\frac{x-4}{3}=\frac{2y-2}{-4}=z-1$ are coplanar.

Q.9- Find the shortest distance between the following lines :
$r=3\hat{\imath}+5\hat{\jmath}+7\hat{k}+\lambda(\hat{\imath}-2\hat{\jmath}+\hat{k})$ and $\vec{r}=(-\hat{\imath}-\hat{\jmath}-\hat{k})+\mu(7\hat{\imath}-6\hat{\jmath}+\hat{k})$.

3D→RACE-3

Q.1- Distance of the point (α, β, γ) from y axis is:

(A) β (B) $|\beta|$ (C) $|\beta| + |\gamma|$ (D) $\sqrt{\alpha^2 + \gamma^2}$

Q.2- If the direction cosines of a line are k, k and k, then

(A) k > 0 (B) 0 < k < 1 (C) k = 1 (D) $k = \frac{1}{\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$

Q.3- The direction cosines of the vector $(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$ are.

(A) $-\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$ (B) $-\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$ (C) $-\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$ (D) $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$

Q.4- Two lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x}{5} = \frac{y}{10} = \frac{z}{15}$ are mutually.

(A) perpendicular (B) skew (C) coinciding (D) parallel

Q.5- The vector equation of the line through the points (3, 4, –7) and (1, –1, 6) is

(A) $\vec{r} = 3\hat{\imath} + 4\hat{\jmath} - 7\hat{k} + \lambda(-2\hat{\imath} - 5\hat{\jmath} + 13\hat{k})$ (B) $\vec{r} = 3\hat{\imath} + 4\hat{\jmath} + 7\hat{k} + \lambda(-2\hat{\imath} - 5\hat{\jmath} + 13\hat{k})$

(C) $\vec{r} = 3\hat{\imath} - 4\hat{\jmath} - 7\hat{k} + \lambda(-2\hat{\imath} + 5\hat{\jmath} + 13\hat{k})$ (D) $\vec{r} = 3\hat{\imath} + 4\hat{\jmath} - 7\hat{k} + \lambda(2\hat{\imath} - 5\hat{\jmath} + 13\hat{k})$

Q.6- Find the angle between the lines

$\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k} + \lambda(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ and $\vec{r} = (2\hat{\jmath} - 5\hat{k}) + \mu(6\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$ is

(A) $cos^{-1}\frac{19}{21}$ (B) $cos^{-1}\frac{20}{21}$ (C) $cos^{-1}\frac{21}{19}$ (D) $cos^{-1}\frac{21}{20}$

Q.7- The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are given by.

(A) (0, 5, 7) (B) (0, 5, 0) (C) (2, 0, 0) (D) (0, 0, 7)

Q.8- The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is

(A) parallel to x-axis (B) parallel to y-axis

(C) parallel to z-axis (D) perpendicular to z-axis

Q.9- P is point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of P is 5, then its y co-ordinate is

(A) –1 (B) –2 (C) 1 (D) 2

Q.10- The cartesian equation of the line passing through the points (3, 5, 4) and (5, 8, 11) is.

(A) $\frac{(x-3)}{2} = \frac{(y-5)}{8} = \frac{(z-4)}{11}$ (B) $\frac{(x-3)}{2} = \frac{(y-5)}{8} = \frac{(z-4)}{-11}$

(C) $\frac{(x-3)}{2} = \frac{(y-5)}{-8} = \frac{(z-4)}{11}$ (D) $\frac{(x-3)}{-2} = \frac{(y-5)}{8} = \frac{(z-4)}{11}$

3D→RACE-4

Q.1- Write the direction cosines of a line equally inclined to the three coordinate axes.

Q.2- The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, –2) is 4. Find its z-coordinate.

Q.3- If the cartesian equation of the line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$; Write the vector equation for the line.

Q.4- Find the equation of the line passing through the point (2, –1, 1) and parallel to the line

$\frac{x-3}{2} = \frac{y+1}{7} = \frac{z-2}{-3}$

Q.5- Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

Q.6- Find the vector equation of the line passing through the point (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

Q.7- Find the equation of the perpendicular drawn from the point (2, 4, –1) to the line

$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.

Q.8- Find the shortest distance between the following lines whose vector equations are: $\vec{r}=(1-t)\hat{\imath}+(t-2)\hat{\jmath}+(3-2t)\hat{k}$ and $\vec{r}=(s+1)\hat{\imath}+(2s-1)\hat{\jmath}-(2s+1)\hat{k}$

Q.9- Find the vector and Cartesian equations of the line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$.

Q.10- Find the length and the foot of the perpendicular drawn from the point (2, –1, 5) to the line
$\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.

3D→RACE-5

Q.1- If the direction ratios of a line are 1, 1, 2, find the direction cosines of the line.

Q.2- Find the direction cosines of the line passing through the points P (2, 3, 5) and Q (–1, 2, 4).

Q.3- If a line makes an angle of 30°, 60°, 90° with the positive direction of x, y, z-axes, respectively, then find its direction cosines.

Q.4- The x-coordinate of a point on the line joining the points Q (2, 2, 1) and R (5, 1, –2) is 4. Find its z- coordinate.

Q.5- Find the distance of the point (– 2, 4, – 5) from the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

Q.6- Find the angle between the lines whose direction cosines are given by the equations: $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.

Q.7- Find the co-ordinates of the foot of perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, –1, 3) and C (2, –3, –1).

Q.8- Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

Q.9- The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are given by

(A) (2, 0, 0) (B) (0, 5, 0) (C) (0, 0, 7) (D) (0, 5, 7)

Q.10- P is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of P is 5, then its y co-ordinate is

(A) 2 (B) 1 (C) –1 (D) –2

Q.11- If α, β, γ are the angles that a line makes with the positive direction of x, y, z axis, respectively, then the direction cosines of the line are.

Q.12- The distance of a point P (a, b, c) from x-axis is

(A) $\sqrt{a^2+c^2}$ (B) $\sqrt{a^2+b^2}$ (C) $\sqrt{b^2+c^2}$ (D) b^2+c^2

Q.13- The equations of x-axis in space are

(A) x = 0, y = 0 (B) x = 0, z = 0 (C) x = 0 (D) y = 0, z = 0

Q.14- A line makes equal angles with co-ordinate axis. Direction cosines of this line are

(A) $\pm(1,1,1)$ (B) $\pm\left[\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right]$ (C) $\pm\left[\frac{1}{3},\frac{1}{3},\frac{1}{3}\right]$ (D) $\pm\left[\frac{1}{\sqrt{3}},\frac{-1}{\sqrt{3}},\frac{-1}{\sqrt{3}}\right]$

Q.15- If a line makes angles $\frac{\pi}{2}$, $\frac{3}{4}\pi$ and $\frac{\pi}{4}$ with x, y, z axis, respectively, then its direction cosines are ________

Q.16- If a line makes angles α, β, γ with the positive directions of the coordinate axes, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is ________

Q.17- If a line makes an angle of $\frac{\pi}{4}$ with each of y and z axis, then the angle which it makes with x-axis is __________

Q.18- The points (1, 2, 3), (–2, 3, 4) and (7, 0, 1) are collinear.

Q.19- The vector equation of the line passing through the points (3,5,4) and (5,8,11) is
$\vec{r}=3\hat{\imath}+5\hat{\jmath}+4\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+7\hat{k})$

3D→RACE-6

Short Answer

Q.1- Find the position vector of a point A in space such that $\overrightarrow{OA}$ is inclined at 60° to OX and at 45° to OY and $|\overrightarrow{OA}| = 10$ units.

Q.2- Find the vector equation of the line which is parallel to the vector $3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$ and which passes through the point (1,–2,3).

Q.3- Show that the lines
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also, find their point of intersection.

Q.4- Find the angle between the lines
$\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k} + \lambda(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ and $\vec{r} = (2\hat{\jmath} - 5\hat{k}) + \mu(6\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$

Q.5- Prove that the line through A (0, –1, –1) and B (4, 5, 1) intersects the line through C (3, 9, 4) and D (– 4, 4, 4).

Q.6- Prove that the lines x = py + q, z = ry + s and x = p′y + q′, z = r′y + s′ are perpendicular if pp′ + rr′ + 1 = 0.

Q.7- Find the equations of the two lines through the origin which intersect the line
$\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at angles of $\frac{\pi}{3}$ each.

Q.8- Find the angle between the lines whose direction cosines are given by the equations $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$.

Long Answer

Q.9- Find the foot of perpendicular from the point (2,3,–8) to the line
$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line.

Q.10- Find the distance of a point (2,4,–1) from the line
$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

Q.11- Find the shortest distance between the lines given by $\vec{r} = (8 + 3\lambda\hat{\imath} - (9 + 16\lambda)\hat{\jmath} + (10 + 7\lambda)$ and $\vec{r} = 15\hat{\imath} + 29\hat{\jmath} + 5\hat{k} + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$.

Q.12- $\overrightarrow{AB} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\overrightarrow{CD} = -3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ are two vectors. The position vectors of the points A and C are $6\hat{\imath} + 7\hat{\jmath} + 4\hat{k}$ and $-9\hat{\jmath} + 2\hat{k}$, respectively. Find the position vector of a point P on the line AB and a point Q on the line CD such that $\overrightarrow{PQ}$is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ both.

Q.13- Show that the straight lines whose direction cosines are given by 2l + 2m – n = 0 and mn + nl + lm = 0 are at right angles.

Q.14- Show that the straight lines whose direction cosines are given by 2l + 2m – n = 0 and mn + nl + lm = 0 are at right angles.

Objective Type Questions Choose the correct answer from the given four options in each of the Exercises from 15 to 17.

Q.15- Distance of the point (α, β, γ) from y-axis is

(A) β (B) $|\beta|$ (C) $|\beta| + |\gamma|$ (D) $\sqrt{\alpha^2 + \gamma^2}$

Q.16- If the directions cosines of a line are k,k,k, then

(A) k>0 (B) 0<k<1 (C) k=1 (D) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

Q.17- The area of the quadrilateral ABCD, where A(0,4,1), B (2,3,–1), C(4, 5, 0) and D (2,6,2), is equal to

(A) 9 sq. units (B) 18 sq. units (C) 27 sq. units (D) 81 sq. units

Fill in the blanks in each of the Exercises 18 to 19.

Q.18- The direction cosines of the vector $(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$ are __________.

Q.19- The vector equation of the line through the points (3,4,–7) and (1,–1,6) is __________.

Relations and Functions→RACE-1

Q.1- Let A = {1, 2, 3, 4} and B = {x, y, z}. Consider the subset R = {(1, x), (1, y), (2, z), (3, x)} of A × B. Is R, a relation from A to B? If yes, find domain and range of R. Draw arrow diagram of R.

Q.2- If A = {1, 2, 3, 4} and B = {a, b, c}, then find the domain and range of the relation R = {(1, a), (2, b), (3, a)} from A to B.

Q.3- If A = {1, 2, 3}, then determine the reflexive, symmetric and transitive relations on A.
(a) R_1 = {(1, 2), (2, 1)} (b) R_2 = {(1, 1), (2, 2), (3, 3), (1, 2)}
(c) R_3 = {(1, 2), (2, 1), (1, 1), (2, 2)} (d) R_4 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

Q.4- Let L be the set of all lines in XY-plane and R be the relation in L defined as R = {(L_1, L_2) : L_1 is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.

Q.5- Show that the relation R in R defined as R = {(a, b) : a ≤ b}, is reflexive and transitive but not symmetric.

Q.6- Let N denote the set of all natural numbers and R be the relation on N × N by (a, b) R (c, d) ⇔ ad (b + c) = bc (a + d). Check whether R is an equivalence relation on N × N.

Q.7- If the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 15}, given by R = {(a,b) : a, b ∈ Z}, |a–b| is multiple of 5} is an equivalence relation, then find the equivalence class [2].

Q.8- If A = {1, 2, 3, ..., 9} and R be the relation in A × A defined by (a, b) R (c, d) if a + d = b + c for a, b, c, d ∈ A is an equivalence relation, then find the equivalence class [(2,5)].

Q.9- Let f : R → R be defined by f(x) = x^2 + 1, then find the pre-images of 17 and –3.

Q.10- The domain of the function f : R→ R defined by $f(x) = \sqrt{x^2 - 3x + 2}$ is ?

Q.11- Let f : R → R be the function defined by $f(x) = \frac{1}{2-cosx}; x \in R$, then find the range of f.

Q.12- Let A ∈ R – {3} and B ∈ R – {1}. Consider the function f : A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$.

Relations and Functions→RACE-2

Q.1- Consider $f: R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective.

Q.2- Let A = {x ∈ Z : 0 ≤ x ≤ 12}. Show that R = {(a, b) : a, b ∈ A, |a – b| is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. Also; write the equivalence class [2].

Q.3- Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}, \forall x \in R$ is neither one-one nor onto.

Q.4- Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R {(a, b) : b = a + 1} is reflexive, symmetric or transitive.

Q.5- The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1), (1, 1)} is
(A) symmetric and transitive, but not reflexive
(B) reflexive and symmetric, but not transitive
(C) symmetric, but neither reflexive nor transitive
(D) an equivalence relation

Q.6- Let A = {1, 3, 5}. Then the number of equivalence relations in A containing (1, 3) is:
(A) 1 (B) 2 (C) 3 (D) 4

Q.7- Check whether the relation R in the set N of natural numbers given by R = {(a, b) : a is divisor of b} is reflexive, symmetric or transitive. Also, determine whether R is an equivalence relation.

Q.8- Write the smallest reflexive relation on set A = {a, b, c}.

Q.9- (a) Check whether the relation R defined on the set {1, 2, 3, 4} as R = {(a, b) ; b = a + 1} is transitive. Justify your answer.

Q.10- If the relation R on the set A = {x : 0 ≤ x ≤ 12} given by R = {(a, b) : a = b} is an equivalence relation, then find the set of all elements related to 1.

Q.11- Let set X = {1, 2, 3} and a relation R is defined in X as R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are?

(A) {(1, 1), (2, 3), (1, 2)} (B) {(3, 3), (3, 1), (1, 2)}
(C) {(1, 1), (3, 3), (3, 1), (2, 3)} (D) {(1, 1), (3, 3), (3, 1), (1, 2)}

Q.12- If $R = \{x, y\}; x, y \in Z, x^2 + y^2 \leq 4$ is a relation in set Z, then domain of R is :

(A) {0, 1, 2} (B) {–2, –1, 0, 1, 2} (C) {0, –1, –2} (D) {–1, 0, 1}

Q.13- Let $\mathrm{X} = \{\mathrm{x}^2 : \mathrm{x} \in \mathrm{N}\}$ and the function $\mathrm{f}: \mathrm{N} \to \mathrm{X}$ is defined by $\mathrm{f(x)} = \mathrm{x}^2, \mathrm{x} \in \mathrm{N}$. Then, this function is :

(A) injective only (B) not bijective (C) surjective only (D) bijective

Q.14- A function f : R→R defined by $f(x) = 2 + x^2$ is:

(A) not one-one (B) one-one (C) not onto (D) neither one-one nor onto

Relations and Functions→RACE-3

Q.1- Consider the relation R = {(a, b) (a, c), (a, a), (c, c)} on the set A = {a, b, c, d}. Minimum number of elements of A × A which must be adjoined to R in order to make R an equivalence relation is:

(A) 4 (B) 5 (C) 6 (D) 7

Q.2- If A and B are two sets having 10 and 15 elements respectively and 8 elements are common in A and B; then number of relations which can be defined from A to B are:

(A) 2^{17} (B) 2^{25} (C) 2^{150} (D) 2^{80}

Q.3- Let R be a relation on natural number N defined as aRb ⇔ a is a factor of b; then relation R is

(A) Reflexive and symmetric (B) Reflexive and transitive
(C) Symmetric and transitive (D) Only reflexive

Q.4- Let $f(x) = |lx - 1l - 1|.\ f: [0,2] \to R$, Range of function is:

(A) [0, 2] (B) [0, 1] (C) R (D) None

Q.5- Let f, g: R →R. If f and g are bijective, then f + g is:

(A) Always bijective (B) May not be bijective
(C) Must be injective (D) Must be surjective

Q.6- The number of reflexive relations defined on the set S = {1, 2, 3, 4, 5} must be:

(A) 25 (B) 225 (C) 52 (D) 220

Q.7- Which of the following is a function ?

(A) {(2, 1) , (2, 2), (2, 3), (2, 4)} (B) {(1, 4), (2, 5), (1, 6), (3, 9)}
(C) {(1, 2), (3, 3), (2, 3), (1, 4)} (D) {(1, 2), (2, 2), (3, 2), (4, 2)}

Q.8- If X = {a, b, c, d, e} and Y = {p, q, r, s, t}; then which of the following subset(s) of XY is a function from X to Y?

(A) {(a, r) (b, r) (b, s) (d, t) (e, q) (c, q)} (B) {(a, r) (b, p) (c, t) (d, q)}
(C) {(a, p) (b, t) (c, r) (d, s) (e, q)} (D) None of these

Q.9- If A = {a, b} and B = {0, 1, 2}; then number of functions defined from A to B is:

(A) 9 (B) 8 (C) 6 (D) None of these

Q.10- Let $f: R \rightarrow R$ be defined as $f(x) = \begin{cases} 2x, & if \quad x > 3 \\ x^2, & if \quad 1 < x \leq 3; \\ 3x & if \quad x \leq 1 \end{cases}$ then the value of f(–1)+f(2)+f(4) is:

(A) 9 (B) 120 (C) 0 (D) None of these

Q.11- For a given function to be defined, the domain of the function $f(x) = \sqrt{log\left(\frac{5x-x^2}{6}\right)}$ is:

(A) (2, 3) (B) [2, 3] (C) [0, 3] (D) [0, 2]

Q.12- Domain of $2^x + 2^y = 2$ is:

(A) (–∞, 1] (B) (0,1) (C) (–∞, –1) (D) (–∞, 1)

Q.13- Let A = {2, 3, 4, 5 .... 17}. Let ≅ be the equivalence relation on A × A, cartesian product of A with itself, defined by (a, b) ≅ (c, d) iff ad = bc. Then, the number of ordered pairs of the; equivalence class of (3, 2) is :

(A) 6 (B) 5 (C) 7 (D) 4

Q.14- Domain of f(x) = log |logx| is:

(A) (0,1) ∪ [1, ∞) (B) (0,1] ∪ [1, ∞) (C) (0,1] ∪ (1, ∞) (D) (0,1] ∪ (1, ∞)

Q.15- Range of (–1)[x], (when [.] denotes greatest integer function) is :

(A) {–1, 0} (B) {–1, 1} (C) {0, 1} (D) None of these

CASE STUDY – I

Q.16- Consider the function $f(x) = \frac{ax+b}{cx+d}$, where $x \neq -\frac{d}{c}$,

Based on the above information, answer the following:

(i) f(x) is one-one if

(A) ab – cd≠ 0 (B) ad – bc ≠0 (C) (C) ab + cd ≠ 0 (D) None of these

(ii) f(x) is onto if :

(A) if a ≠ c (B) b ≠ d (C) ad + bc ≠ c (D) None of these

(iii) The range of the function $\frac{x-1}{x^2-3x+2}$ must be:

(A) All reals except 0 and 1 (B) All reals except –1 and 1

(C) All reals except 0 and –1 (D) None of these

(iv) The function f(x) is:-

(A) Even function (B) Odd function

(C) Nothing can be said (D) None of these

(v) The value of f(–x) is :-

(A) $\frac{ax-b}{-cx+d}$ (B) $\frac{b-ax}{d-cx}$ (C) $\frac{ax-b}{cx+d}$ (D) $-\left(\frac{ax+b}{cx+d}\right)$

CASE STUDY – II

Q.17- An organization conducted bike race under two different categories-boys and girls. Totally there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let B = {b_1, b_2, b_3}; G = {g_1, g_2} where B represents the set of boys selected and G the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions.

Based on the above information, answer the following:

(i) Ravi wishes to form all the relations possible from B to G. How many such relations are possible?

(A) 2^6 (B) 2^5 (C) 0 (D) 2^3

(ii) Let R : B→ B be defined by R = {(x, y): x and y are students of same sex}, Then this relation R is _________

(A) Equivalence

(B) Reflexive only

(C) Reflexive and symmetric but not transitive

(D) Reflexive and transitive but not symmetric

(iii) Ravi wants to know among those relations, how many functions can be formed from B to G?

(A) 2^2 (B) 21^2 (C) 3^2 (D) 2^3

(iv) Let $R: B \to G$ be defined by $R = \{(b_1, g_1), (b_2, g_2)(b_3, g_1)\}$, then R is

(A) Injective (B) Surjective

(C) Neither Surjective nor Injective (D) Surjective and Injective

(v) Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?

(A) 0 (B) 2! (C) 3! (D) 0!

Relations and Functions→RACE-4

Q.1- Show that the number of equivalence relations on the set {1,2,3} containing (1,2) and (2,1) is two.

Q.2- Let R = {(1,3), (4,2), (2,4), (2,3), (3,1)} be a relation on the set A = {1, 2, 3, 4}. Check whether R is reflexive, symmetric or transitive.

Q.3- Let A = {1, 2, 3}, B {4, 5, 6, 7} and Let f = {(1,4), (2, 5), (3, 6)} be a function from A to B. Check f(x) is bijection or not.

Q.4- Let R be the equivalence relation in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a –b)}. Write the equivalence class [0].

Q.5- Show that the function $f: R \to \{x \epsilon R: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one and onto function.

Q.6- Show that the relation R in R defined as R = {(a, b) : a ≤ b}, is reflexive and transitive but not symmetric.

Q.7- Let N denote the set of all natural numbers and R be the relation on N × N given by (a, b) R (c, d) ⇔ ad(b + c) = bc(a + d). Check whether R is an equivalence relation on N × N.

Q.8- Show that the relation R on the set $A = \{x \in Z: 0 \leq x \leq 12\}$, given by $R = \{(a, b): |a - b|$ is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1 i.e. equivalence class [1].

Q.9- Let A = R – {3} and B = R – {1}. Consider the function f : A →B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto ? Justify your answer.

Q.10- Show that the function f : R → R given by f(x) = ax + b, where a,b $\in R, a \neq 0$ is a bijection.

Inverse Trigonometric Functions→RACE-1

Q.1- Evaluate: $Sign\left(tan^{-1}\frac{15}{8}\right)$

Q.2- Evaluate: $cos\left[sin^{-1}\frac{1}{4}+sec^{-1}\frac{4}{3}\right]$.

Q.3- Evaluate: $tan^{-1}\sqrt{3}-sec^{-1}(-2)$.

Q.4- Evaluate: $sin^{-1}\left(sin\frac{7\pi}{6}\right)$.

Q.5- Express in the simplest form : $tan^{-1}\left(\frac{cosx-sinx}{cosx+sinx}\right), -\frac{\pi}{4}<x<\frac{\pi}{4}$.

Q.6- If $sin\left(sin^{-1}\frac{1}{5}+cos^{-1}x\right)=1$, then find the value of x.

Inverse Trigonometric Functions→RACE-2

Q.7- If $tan^{-1}\frac{x-3}{x-4}+tan^{-1}\frac{x+3}{x+4}=\frac{\pi}{4}$, then find the value of x.

Q.8- Find the value of $tan^{-1}\sqrt{3}-cot^{-1}(-\sqrt{3})$.

Q.9- Prove that: $3sin^{-1}x=sin^{-1}(3x-4x^3), x\in\left[-\frac{1}{2},\frac{1}{2}\right]$

Q.10- Solve for x: $tan^{-1}(x+1)+tan^{-1}(x-1)=tan^{-1}\left(\frac{8}{31}\right)$.

Q.11- $tan^{-1}3+tan^{-1}\lambda=tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$ is valid for what values of λ?

(A) $\lambda\in\left(-\frac{1}{3},\frac{1}{3}\right)$ (B) $\lambda>\frac{1}{3}$ (C) $\lambda>\frac{1}{3}$ (D) All real values of λ

Q.12- The range or the principal value branch of the function $y=sec^{-1}x$ is ………… .

Q.13- The principal value of $cos^{-1}\left(-\frac{1}{2}\right)$ is ………… .

Q.14- Prove that $tan^{-1}\left(\frac{1}{4}\right)+tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}sin^{-1}\left(\frac{4}{5}\right)$

Q.15- Simplify $sec^{-1}\left(\frac{2}{2x^2-1}\right), 0<x<\frac{1}{\sqrt{2}}$.

Q.16- The principle value of $cos^{-1}\left(\frac{1}{2}\right)+sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is:

(A) $\frac{\pi}{12}$ (B) π (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Q.17- The principle value of $tan^{-1}\left(tan\frac{9\pi}{8}\right)$ is:

(A) $\frac{\pi}{8}$ (B) $\frac{3\pi}{8}$ (C) $-\frac{\pi}{8}$ (D) $-\frac{3\pi}{8}$

Q.18- What is the domain of the function $cos^{-1}(2x-3)$?

Q.19- The principle value of $[tan^{-1}\sqrt{3}-cot^{-1}(-\sqrt{3})]$ is :

(A) π (B) $-\frac{\pi}{2}$ (C) 0 (D) $2\sqrt{3}$

Inverse Trigonometric Functions→RACE-3

Q.20- The principle value of $sec^{-1}(-2)$ is?

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Q.21- The domain of $sin^{-1}x + cosx$ is:

(A) $[-1, \infty)$ (B) $(-1, 1)$ (C) $[-1, 1]$ (D) $(\infty, 1]$

Q.22- The principle value of $cos^{-1}\left(-\frac{1}{2}\right)$ is?

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $-\frac{2\pi}{3}$ (D) $\frac{\pi}{6}$

Q.23- The principle value of $cos^{-1}\left\{sin\left(cos^{-1}\frac{1}{2}\right)\right\}$ is?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Q.24- The principle value of $cot[sin^{-1}\{cos(tan^{-1}1)\}]$ is:

(A) $\frac{1}{2}$ (B) ∞ (C) $\frac{1}{\sqrt{2}}$ (D) 1

Q.25- The domain of $sec^{-1}(2x + 1)$ is:

(A) $(-\infty,-1) \cup [0, \infty)$ (B) $(-\infty,-1) \cup (0, \infty)$

(C) $(-\infty,-1] \cup [0, \infty)$ (D) None of these

Q.26- $Sec\left[90^{o} - cot^{-1}\left(\frac{1}{3}\right)\right]$ is equal to?

(A) $\sqrt{10}$ (B) $\frac{1}{3}$ (C) 3 (D) $\frac{\sqrt{10}}{3}$

Q.27- Domain of $f(x) = sin^{-1}(-x^2)$ is:-

(A) $(-1, 1)$ (B) $(-\infty, 1]$ (C) $[-1, \infty]$ (D) $(-1, 1)$

Q.28- Domain of $cos^{-1}(2x - 1)$ is:-

(A) $[0, 1]$ (B) $(0, 1)$ (C) $(0, 1]$ (D) $[0, 1)$

Q.29- The Principal value of $tan^{-1}\left[2sin\left(2cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ is?

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

Case Study-I

Q.30- Given; $\alpha = cos^{-1}\left(\frac{3}{5}\right)$ and $\beta = tan^{-1}\left(\frac{2}{3}\right)$, where $0 < \alpha, \beta < \frac{\pi}{2}$.

Based on the above information, answer the following questions:

(i) $(\alpha + \beta)$ is equal to:

(A) $tan^{-1}(16)$ (B) $tan^{-1}(18)$ (C) $tan^{-1}(9)$ (D) None of these

(ii) The range of α is:

(A) $[0, \pi]$ (B) $(0, \pi)$ (C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) None of these

(iii) $(\alpha - \beta)$ is equal to:

(A) $tan^{-1}\left(\frac{6}{53}\right)$ (B) $cos^{-1}\left(\frac{17}{5\sqrt{13}}\right)$ (C) $sin^{-1}\left(\frac{17}{5\sqrt{13}}\right)$ (D) None of these

(iv) $cos^{-1}\left(-\frac{7}{25}\right)$ is equal to:

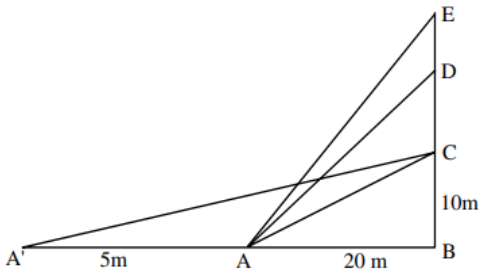
(A) α (B) $\frac{\alpha}{2}$ (C) $4\,\alpha$ (D) $2\,\alpha$

(v) The principal value branch of angle β is :

(A) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ (B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ (C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (D) None of these

Case Study-II

Q.31- The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. "A" is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. "C" is at the height of 10 metres from the ground level. For the viewer A, the angle of elevation of "D" is double the angle of elevation of "C". The angle of elevation of "E" is triple the angle of elevation of "C" for the same viewer. **Look at the figure given and based on the above information, answer the following questions:**



(i) Measure of $\angle CAB$ =?

(A) $tan^{-1}(2)$ (B) $tan^{-1}\left(\frac{1}{2}\right)$ (C) $tan^{-1}(1)$ (D) $tan^{-1}(3)$

(ii) Measure of $\angle DAB$ =?

(A) $tan^{-1}\left(\frac{3}{4}\right)$ (B) $tan^{-1}(3)$ (C) $tan^{-1}\left(\frac{4}{3}\right)$ (D) $tan^{-1}(4)$

(iii) Measure of $\angle EAB$ =?

(A) $tan^{-1}(11)$ (B) $tan^{-1}(3)$ (C) $tan^{-1}\left(\frac{2}{11}\right)$ (D) $tan^{-1}\left(\frac{11}{2}\right)$

(iv) A' is another viewer standing on the same line of observation across the road. If the width of the road is 5 meters, then the difference between ∠CAB and ∠CA'B is?

(A) $tan^{-1}\left(\frac{1}{2}\right)$ (B) $tan^{-1}\left(\frac{1}{12}\right)$ (C) $tan^{-1}\left(\frac{2}{5}\right)$ (D) $tan^{-1}\left(\frac{11}{21}\right)$

(v) Domain and Range of $tan^{-1}x$ =?

(A) $R^{+}; \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (B) $R^{-}; \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (C) $R; \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (D) $R; \left(0, \frac{\pi}{2}\right)$

Inverse Trigonometric Functions→RACE-4

Q.32- Find the principal value of $tan^{-1}\left\{sin\left(-\frac{\pi}{2}\right)\right\}$?

Q.33- Which is greater, tan1 or $\tan^{-1} 1$?

Q.34- Find the domain of $f(x) = sin^{-1}(-x^2)$.

Q.35- Find the domain of $f(x) = sin^{-1}x + cosx$.

Q.36- Find the principal value of $cos^{-1}\left\{sin\left(cos^{-1}\frac{1}{2}\right)\right\}$?

Q.37- Find the value of $sin^{-1}\left[cos\left\{sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}\right]$?

Q.38- For the principal values, evaluate the following:

(i) $cot^{-1}(-1) + cosec^{-1}(-\sqrt{2}) + sec^{-1}(2)$

(ii) $cot^{-1}(-\sqrt{3}) + tan^{-1}(1) + sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Q.39- (i) Find the domain of $sec^{-1}(2x + 1)$

(ii) Find the principal value of $sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ and $sec^{-1}(-2)$.

Q.40- For the principal values, evaluate each of the following :

(i) $tan^{-1}\left\{2cos\left(2sin^{-1}\frac{1}{2}\right)\right\}$ (ii) $cot[sin^{-1}\{cos(tan^{-1}1)\}]$

Q.41- Find the value of the expression $sin[cot^{-1}\{cos(tan^{-1}1)\}]$.

Q.42- If $x = cosec\left[tan^{-1}\left\{cos\left(cot^{-1}\left(sec(sin^{-1}a)\right)\right)\right\}\right]$

and $y = sec\left[cot^{-1}\left\{sin\left(tan^{-1}\left(cosec(cos^{-1}a)\right)\right)\right\}\right]$;

where a ∈ [0,1], then find the relationship between x and y in terms of a

Matrices →RACE-1

Q.1- Find x, y, z and w if $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.

Q.2- If $A = \begin{bmatrix} sin\theta & -cos\theta \\ cos\theta & sin\theta \end{bmatrix}$ and $B = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ then compute $(sin\theta)A + (cos\theta)B$.

Q.3- (i) Find x, y if $3\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} - 2\begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} x & -4 \\ 3 & y \end{bmatrix} = 0$

(ii) Find the Matrix X such that 2A + B + X = O, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$.

Q.4- Find A and B, if $2A + 3B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 6 & 2 \end{bmatrix}$.

Q.5- Let $A = \begin{bmatrix} 0 & -tan\frac{\alpha}{2} \\ tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I be the identity matrix of order 2.

Show that $I + A = (I - A)\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$.

Q.6- Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = O$. Use this result to find A^5.

Q.7- If $A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 2 & 0 \end{bmatrix}$, prove that (AB)′=B′A′.

Q.8- Express $\begin{bmatrix} 2 & 4 & -1 \\ 3 & 5 & 8 \\ 1 & -2 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Matrices →RACE-2

Q.9- Find matrix A such that $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$.

Q.10- If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.

Q.11- If A = $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, show that (A – 2I) (A – 3I) = O.

Q.12- Given a skew-symmetric matrix A = $\begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$, the value of $(a + b + c)^2$ is …… .

Q.13- If A = $\begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find scalar k so that $A^2 + I = kA$.

Q.14- Find the order of the matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

Q.15- If $B = \begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix}$, find matrix A.

Q.16- If $A = [1 \quad 0 \quad 4]$ and $B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ find AB.

Q.17- If the matrix $A = \begin{bmatrix} 0 & 6-5x \\ x^2 & x+3 \end{bmatrix}$ is symmetric

Find the value of x.

Q.18- The number of all possible matrices of order 2 × 3 with each entry 1 or 2 is :

(A) 16 (B) 6 (C) 64 (D 24

Q.19- If a matrix A is both symmetric and skew-symmetric, then A is necessarily a :

(A) Diagonal matrix (B) Zero square matrix

(C) Square matrix (D) Identity matrix

Q.20- If $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of ab – cd is :

(A) 4 (B) 16 (C) –4 (D) –16

Q.21- For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^r = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ $P - Q$ is:

(A) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 4 & 3 \\ 0 & -3 \\ -1 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

Q.22- A matrix $A = [a_{ij}]_{3\times 3}$ is defined by

$$a_{ij} = \begin{cases} 2i+3j & ; \quad i<j \\ 5 & ; \quad i=j \\ 3i-2j & ; \quad i>j \end{cases}$$

The number of elements in A which are more than 5, is :

(A) 3 (B) 4 (C) 5 (D) 6

Q.23- For the matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $(X^2 - X)$ is:

(A) 2I (B) 3I (C) I (D) 5I

Matrices →RACE-3

Q.24- If X = $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ and 3X – $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ then 'a' is equal to-

(A) 1 (B) 2 (C) 0 (D) –2

Q.25- If A = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and a and b are arbitary constants then $(aI + bA)^2 =$

(A) $a^2I + abA$ (B) $a^2I + 2abA$ (C) $a^2I + b^2A$ (D) None of these

Q.26- If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$, then B equals

(A) I cos θ + J sin θ (B) I cos θ – J sin θ

(C) I sin θ + J cos θ (D) –I cos θ + J sin θ

Q.27- If $[1\ x\ 2] \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 2 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -1 \end{bmatrix} = 0$, then the value of x is-

(A) –1 (B) 0 (C) 1 (D) 2

Q.28- If $A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$ then element a_{21} of A^2 is-

(A) 22 (B) –15 (C) –10 (D) 7

Q.29- If $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ then

(A) x = 2, y = 1 (B) x = 1, y = 2 (C) x = 3, y = 2 (D) x = 2, y = 3

Q.30- If $A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ then $B^T A^T$ is equal to-

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Q.31- If $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ then $(AB)^T$ equals:

(A) $\begin{bmatrix} 5 & 9 \\ 4 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & 9 \\ 16 & 12 \end{bmatrix}$ (C) $\begin{bmatrix} 5 & 16 \\ 9 & 16 \end{bmatrix}$ (D) $\begin{bmatrix} 5 & 9 \\ 12 & 16 \end{bmatrix}$

Q.32- If $\begin{bmatrix} 1 & 0 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & -2 \end{bmatrix}$, then values of a, b are-

(A) 1, –2 (B) –1, 2 (C) –1, –2 (D) 1, 2

Q.33- A matrix $A = [a_{ij}]$ of order 2×3whose elements are such that $a_{ij} = i + j$ is-

(A) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 5 & 4 \end{bmatrix}$ (D) None of these

Q.34- If X and Y two matrices are such that $X - Y = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $X + Y = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ then Y matrix is

(A) $\begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix}$ (D) None of these

Q.35- If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, then skew symmetric part of A is :-

(A) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{-1}{2} & -1 \\ -2 & \frac{-3}{2} \end{bmatrix}$

Case Study-I

Q.36- Three schools DPS, CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs.100 and Rs. 50 each respectively. The numbers of articles sold are given as



School/Article	DPS	CVC	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the above information answer the following:

(i) What is the total money (in Rupees) collected by the school DPS?

(A) 700 (B) 7000 (C) 6125 (D) 7875

(ii) What is the total amount of money (in Rs.) collected by schools CVC and KVS?

(A) 14000 (B) 15725 (C) 21000 (D) 13125

(iii) What is the total amount of money collected by all three schools DPS, CVC and KVS?

(A) Rs. 15775 (B) Rs. 14000 (C) Rs. 21000 (D) Rs. 17125

(iv) If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?

(A) Rs. 18,000 (B) Rs. 6,750 (C) Rs. 5,000 (D) Rs. 21,250

(v) How many articles (in total) are sold by three schools?

(A) 230 (B) 130 (C) 430 (D) 330

Case Study-II

Q.37- Assume that X, Y, Z, W and P are matrices of order 2 × n, 3 × k, 2 × p, n × 3 and p×k; respectively. **Based on the above information; answer the following:**

(i) The restriction on an so that XZ will be defined is :

(A) n = p (B) n = 2 (C) n = 3 (D) n = k

(ii) If n = p, then the order of the matrix 7X – 5Z is :

(A) p × 2 (B) 2 × n (C) n × 3 (D) p × n

(iii) The restriction on n, k and p so that PY+WY will be defined are:

(A) k = 3, p = n (B) k is arbitrary, p = 2

(C) p is arbitrary, k = 3 (D) k = 2, p = 3

(iv) If p = 3, then the order of the matrix 3Y + 10P is :

(A) n × 3 (B) 2 × k (C) 3 × k (D) k × p

(v) The multiplication of matrices W and P is defined if and only if :

(A) p = 3 (B) n = p (C) n = k (D) k = 3

Matrices →RACE-4

Q.38- If matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$, then the write the value of λ.

Q.39- Find the value of x + y from the following equation :

$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Q.40- Find x, y, z and w if $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.

Q.41- Construct a matrix $A = [a_{ij}]$ of order 2 × 3, in which elements are $a_{ij} = \frac{1}{2}|2i - 3j|$.

Q.42- If $\begin{bmatrix} a+b & 2 \\ 7 & ab \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 7 & 8 \\ -3 & 4 \end{bmatrix}$, then find the value of a and b.

Q.43- If $A = \begin{bmatrix} 4 & 2 & -5 \\ 1 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -7 & 0 \\ -1 & 2 & 5 \\ 1 & 0 & 2 \end{bmatrix}$ then, prove that: $(AB)^T = B^T A^T$.

Q.44- If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $f(A) = A^2 - 5A + 7I$ then, find f(A).

Q.45- Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Q.46- Find the value of x, if $[1 \quad x \quad 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

Q.47- Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}, C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that CD – AB = O.

Determinants →RACE-1

Q.1- Find the value of $A = \begin{vmatrix} 6 & -7 & 8 \\ 1 & -3 & 1 \\ 2 & 1 & -4 \end{vmatrix}$.

Q.2- Find the area of the triangle whose vertices are $A(at_1^2, 2at_1), B(at_2^2, 2at_2)$ and $C(at_3^2, 2at_3)$.

Q.3- Find the minors and the cofactors of each entry of the third row of the determinant A and hence evaluate det A.

$A = \begin{vmatrix} 6 & -7 & 8 \\ 1 & -3 & 1 \\ 2 & 1 & -4 \end{vmatrix}$

Q.4- Find the adjoint of the matrix $A = \begin{bmatrix} 4 & -6 & 1 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{bmatrix}$ and verify that A(adj A) A=$|A|I_3$.

Q.5- If $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$.

Q.6- Solve using matrix method, for x, y and z :

2x – y – z = 7

3x + y – z = 7

x + y – z = 3.

Determinants →RACE-2

Q.7- If for any 2 × 2 square matrix A, $A(adjA) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

Q.8- If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

Q.9- If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Hence using A^{-1} solve the system of equations

2x – 3y + 5z = 11, 3x + 2y – 4z = –5, x + y – 2z = –3.

Q.10- Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

Q.11- If A is a square matrix satisfying A'A = I, write the value of |A|.

Q.12- Show that for the matrix $\mathrm{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, $\mathrm{A}^3 - 6\mathrm{A}^2 + 5\mathrm{A} + 11\,\mathrm{I} = 0$. Hence, find A^{-1}.

Q.13- If A is a non-singular square matrix of order 3 such that A^2 =3A, then value of |A| is

(A) –3 (B) 3 (C) 9 (D) 27

Q.14- If $\mathrm{A} = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

5x – y + 4z = 5

2x + 3y + 5z = 2

5x – 2y + 6z = –1

Q.15- If A is square matrix of order 3 such that $\mathrm{A(AdjA)} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then find |A|.

Q.16- If $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, find A^{-1}

Hence, Solve the following system of equations.

3x + 4y + 2z = 8

0x + 2y – 3z = 3

x – 2y + 6z = –2

Q.17- If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Q.18- Three points P(2x, x + 3), Q(0, x) and R(x + 3, x + 6) are collinear, then x is equal to :

(A) 0 (B) 2 (C) 3 (D) 1

Q.19- If C_{ij} denote the cofactor of element P_{ij} of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$ then the value of $C_{31}.C_{23}$ is.

(A) 5 (B) 24 (C) –24 (D) –5

Q.20- The system of linear equations

5x + ky = 5,

3x + 3y = 5 ;

will be consistent if

(A) k ≠ –3 (B) k = –5 (C) k = 5 (D) k ≠ 5

Q.21- If A is a square matrix of order 3 and |A| = –5, then |adj A| is :

(A) 125 (B) –25 (C) 25 (D) ±25

Q.22- If for the matrix $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is :

(A) ±3 (B) –3 (C) ±1 (D) 1

Q.23- Let matrix $X = [x_{ij}]$ is given by $X = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$. Then the matrix $Y = [m_{ij}]$, where m_{ij} =minor of x_{ij} is:

(A) $\begin{bmatrix} 7 & -5 & -3 \\ 19 & 1 & -11 \\ -11 & 1 & 7 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & -19 & -11 \\ 5 & -1 & -1 \\ 3 & 11 & 7 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 19 & -11 \\ -3 & 11 & 7 \\ -5 & -1 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 7 & 19 & -11 \\ 1 & 1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

Q.24- If $x = -4$ is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of the other two roots is :

(A) 4 (B) –3 (C) 2 (D) 5

Q.25- The inverse of the matrix $X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is:

(A) $24\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$ (B) $\frac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

Determinants →RACE-3

Q.26- The value of minor and cofactor of 5 in the determinant $\begin{vmatrix} 1 & 0 & 8 \\ 3 & 5 & 9 \\ 4 & 1 & -2 \end{vmatrix}$ is:-

(A) 34, 34 (B) 34, – 34 (C) –34, 34 (D) –34, – 34

Q.27- If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, then the value of $|A^2 - 2A|$ is

(A) 25 (B) –25 (C) 0 (D) 5

Q.28- If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ then $|3A|$ is equal to:

(A) 9 |A| (B) 81 |A| (C) 27 |A| (D) 31 |A|

Q.29- The area of triangle with vertices (5, 4), (–2, 4) and (2, –6) is :-

(A) 70 sq. units (B) 140 sq. units (C) $\frac{35}{2}$ sq. units (D) 35 sq. units

Q.30- A triangle whose area is 3 sq. units and its vertices are (1, 3), (0, 0) and (k, 0) then the value of k is

(A) ± 2 (B) ± 1 (C) ± 4 (D) ± 8

Q.31- If A is an invertible matrix of order 3 and |A| = 5, then value of |adj (A)| is :-

(A) 25 (B) 5 (C) 125 (D) 15

Q.32- If A is an invertible matrix of order 3 such that |A|=2 then value of |adj (adj (A))| is:-

(A) 8 (B) 4 (C) 2 (D) 16

Q.33- The adjoint of the matrix $\begin{bmatrix} -3 & 5 \\ 2 & 0 \end{bmatrix}$ is

(A) $\begin{bmatrix} 0 & 5 \\ 2 & -3 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -5 \\ -2 & -3 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & -5 \\ -2 & 3 \end{bmatrix}$ (D) $\frac{-1}{10}\begin{bmatrix} 0 & -5 \\ -2 & -3 \end{bmatrix}$

Q.34- Inverse matrix of $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$ is-

(A) $-\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (B) $-\frac{1}{8}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ (C) $\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$

Q.35- If A = $\begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$, then adj (adj A) is equal to-

(A) $8\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ (B) $16\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ (C) $64\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ (D) None of these

Q.36- If $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ and M = AB, then M^{-1} is equal to

(A) $\begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 1/6 \end{bmatrix}$ (C) $\begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 1/6 \end{bmatrix}$ (D) $\begin{bmatrix} 1/3 & -1/3 \\ -1/3 & 1/6 \end{bmatrix}$

Case Study-I

Q.37- A and B are square matrices of order 3 given by A = $\begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

Based on the above information, answer the following questions :

(i) The value of |adj(A)| is equal to :

(A) 2^4 (B) 2^5 (C) 2^6 (D) 2^7

(ii) The value of adj adj(A) is equal to :

(A) 2 A (B) 4 A (C) 8 A (D) 16 A

(iii) The value of |adj. adj (B)| is equal to :

(A) 4 (B) 4^2 (C) 4^3 (D) 4^4

(iv) adj(AB) is equal to :

(A) adj(A) (B) adj (B) adj(A) (C) $\frac{adj\ (A)}{adj\ (B)}$ (D) None of these

(v) $|A^2|$ is equal to:

(A) 48 (B) 124 (C) –64 (D) 64

Case Study-II

Q.38- Area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by the determinant :

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since, area is a positive quantity, so we always take the absolute value of the determinant Δ. Also, the area of the triangle formed by three collinear points is zero.

Based on the above information, answer the following questions :

(i) Find the area of the triangle whose vertices are (–2, 6), (3, –6) and (1, 5) :

(A) 30 sq. units (B) 35 sq. units (C) 40 sq. units (D) 15.5 sq. units

(ii) If the points (2, –3) (k, –1) and (0, 4) are collinear, then find the value of 4k :

(A) 4 (B) $\frac{7}{140}$ (C) 47 (D) $\frac{40}{7}$

(iii) If the area of a triangle ABC ; with vertices A(1, 3), B(0, 0) and C(k, 0) is 3 sq. units ; then the value of k is :

(A) 2 (B) 3 (C) 4 (D) 5

(iv) Using determinants, find the equation of the line joining the points A(1, 2) and B (3, 6):

(A) y = 2x (B) x = 3y (C) y = x (D) 4x – y = 5

(v) If A = (11, 7) ; B = (5, 5) and C = (–1, 3) then :

(A) ΔABC is scalene triangle (B) ΔABC is equilateral triangle

(C) A, B and C are collinear (D) None of these

Determinants →RACE-4

Q.39- Find the value of x if the area of triangle is 35 square units with vertices (x, 4), (2, –6) and (5, 4).

Q.40- If A_{ij} is the cofactor of the elements a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32}.A_{32}$.

Q.41- If $\begin{vmatrix} x-1 & x-2 \\ x & x-3 \end{vmatrix} = 0$ then find the value of x.

Q.42- Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Q.43- Determine the values of x for which the matrix A = $\begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & -6 \end{bmatrix}$ is singular.

Q.44- If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that A (adj A) = (adj A) A = |A| I3.

Q.45- If $A = \begin{bmatrix} 1 & tanx \\ -tanx & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} cos2x & -sin2x \\ sin2x & cos2x \end{bmatrix}$.

Q.46- Show that A = $\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$. Hence, find A^{-1}.

Q.47- (i) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, show that $A^{-1} = A^2$.

(ii) If $A = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & -cos\alpha \end{bmatrix}$ is such that $A^T = A^2$, find α.

Q.48- If A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ then, find A^{-1} and solve the following system of equations using it :

x + 2y – 3z = –4, 2x + 3y + 2z = 2, 3x – 3y – 4z = 11